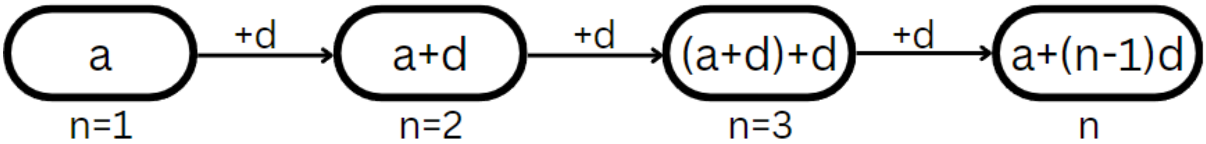
Arithmetic Progression

Arithmetic progression (AP) is a series of numbers where each term is obtained by adding or subtracting a fixed quantity to its preceding term. There is a common difference between each consecutive term.

The first term and the common difference can be positive or negative, whole number, fraction, or decimal values.

Series	**First term (a)**	**Common difference (d)**
3, 5, 7, 9, 11, …	3	2
-24, -17, -10, -3, 4, 11, …	-24	7
50, 45, 40, 35, 30, 25, …	50	-5
$\frac{1}{3}, \frac{8}{15}, \frac{11}{15}, \frac{14}{15}, 1\frac{2}{15}, 1\frac{1}{3}, \ldots$	$\frac{1}{3}$	$\frac{1}{5}$
-0.11, -0.21, -0.31, -0.41, …	-0.11	-0.1



The n[th] term of an AP is obtained by $\boldsymbol{t_n = a + (n-1)d}$

Sum of n terms of an AP is obtained by $\boldsymbol{S_n = \frac{n}{2}[2a + (n-1)d]}$

Arithmetic mean:
When three terms are in AP, the middle term is called the arithmetic mean of the three terms.

If a, b, c are in AP, then b is the arithmetic mean of the AP.

$$b = \frac{a+c}{2}$$

For the AP, 50, 45, 40, 35, 30, 25

45 is the arithmetic mean of 50, 45, 40 $\quad (45 = \frac{50+40}{2})$

40 is the arithmetic mean of 45, 40, 35 $\quad (40 = \frac{45+35}{2})$

35 is the arithmetic mean of 40, 35, 30 $\quad (35 = \frac{40+30}{2})$

Example 1:
Find 5 arithmetic means between 12 and 60
First term a = 12
There are 7 terms. n = 7
7^{th} term is 60. $t_n = 60$
n^{th} term, $t_n = a + (n-1)d$

$t_n = 60 = 12 + (7-1)d = 12 + 6d$
$$d = \frac{60-12}{6} = 8$$
Common difference is 8
Series is 12, 12+8, 20+8, 28+8, 36+8, 44+8, 52+8, 60, or
12, 20, 28, 36, 44, 52, 60

Example 2:
Find the 9^{th} term from the end in the AP 24, 29, 34, 39, …. 109

Hint: Reverse the series
109, … 39, 34, 29, 24

Now a = 109, n = 9, d = -5

$t_n = a + (n-1)d$
$t_{20} = 109 + (9-1)(-5) = 109 + 8(-5) = 69$

Example 3:
If $mt_m = nt_n$, then $t_{(m+n)} = 0$

7 times 7^{th} term of an AP equals 10 times 10^{th} term of the AP, then show that its 17^{th} term is zero.

$t_7 = a + (7-1)d = a + 6d$
$7t_7 = 7a + 42d$

$t_{10} = a + (10-1)d = a + 9d$
$10t_{10} = 10a + 90d$

Given, $7t_7 = 10t_{10}$
$7a + 42d = 10a + 90d$
$3a = -48d$
$a = -16d$

17^{th} term is $t_{17} = a + (17-1)d = a + 16d$

$t_{17} = (-16d + 16d) = 0$

Example 4:
The 8th term of an AP is 10, and its 16th term is 74. Find the series.

$t_8 = 10 = a + (8-1)d = a + 7d$
$t_{16} = 74 = a + (16-1)d = a + 15d$

Two simultaneous linear equations are obtained.
$a + 7d = 10$
$a + 15d = 74$
Solving them, a = -46, d = 8

AP series is -46, -38, -30, -22, -14, -6, 2, **10**, 18, 26, 34, 42, 50, 58, 66, **74**, 82, ...

Example 5:
How many numbers divisible by 8 are present between 245 and 375?

Step 1: Divide 245 by 8 to get 30.625
Step 2: Multiply 8 with (integer+1)
8 x (30+1) = 8 x 31 = 248

Step 3: Divide 375 by 8 to get 46.875
Step 4: Multiply 8 with the integer part
8 x 46 = 368

248 is the first of the series, and 368 is the last of the series

a = 248
t_n = 368
d = 8

$t_n = a + (n-1)d$
$368 = 248 + (n-1)8$

Solving, n = 16
There are 16 numbers divisible by 8 between 245 and 375.

Example 6:
Find if 300 is a term in the series 7, 13, 19, 25, …

a = 7

d = 6
$t_n = a + (n-1)d$

$300 = 7 + (n-1)6$
$n - 1 = \frac{300-7}{6} = \frac{293}{6} = 48\frac{5}{6}$

n is not a whole number. Therefore, 300 is not a term in the series 7, 13, 19, …

Example 7:
The difference between the 25th term and the 15th term is 30. The sum of the 20th term and 8th term is 100. Find the series.

$t_n = a + (n-1)d$

$t_{25} = a + (25-1)d = a + 24d$
$t_{15} = a + 14d$

$t_{25} - t_{15} = 30$
$t_{25} - t_{15} = 30 = 10d$
$d = 3$

$t_{20} = a + 19d$
$t_8 = a + 7d$

$t_{20} + t_8 = 100$
$t_{20} + t_8 = 100 = 2a + 26d = 2a + 26(3) = 2a + 78$
$a = 11$

Series is 11, 14, 17, …

Example 8:
The sum of n terms is $2n^2 + 5n$. Find its nth term.

$S_n = \frac{n}{2}[2a + (n-1)d] = 2n^2 + 5n$

$2a + (n-1)d = 4n + 10$
$2a + dn - d = 4n + 10$
$dn + (2a - d) = 4n + 10$

Comparing LHS and RHS,
$dn = 4n$, or d = 4

$2a - d = 10$, or a = 7

The series is 7, 11, 15, 19, …

The nth term is $t_n = a + (n-1)d = 7 + (n-1)4 = 4n + 3$

Example 9:
The sum of the first 6 terms is 2. The sum of the first 12 terms is 11 1/5. Find the 15th term.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = \frac{6}{2}[2a + 5d] = 6a + 15d = 2$$

$$S_{12} = \frac{12}{2}[2a + 11d] = 12a + 66d = 11\frac{1}{5} = \frac{56}{5}$$

Solving the simultaneous linear equations, d = 1/5, a = -1/6
15th term, $t_{15} = -\frac{1}{6} + 14\left(\frac{1}{5}\right) = 2\frac{19}{30}$

Example 10:
How many terms add up to -58 in the series 0.9, 0.5, 0.1, -0.3, …?

a = 0.9
d = -0.4
S_n = -58

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$-58 = \frac{n}{2}[2(0.9) + (n-1)(-0.4)] = \frac{n}{2}[2.2 - 0.4n] = 1.1n - 0.2n^2$$
$-0.2n^2 + 1.1n + 58 = 0$ This is a quadratic equation
Solving, n = 20 or -29/2. Neglecting the negative value, n = 20.
The sum of 20 terms in the series 0.9, 0.5, 0.1, add up to -58

Summary:

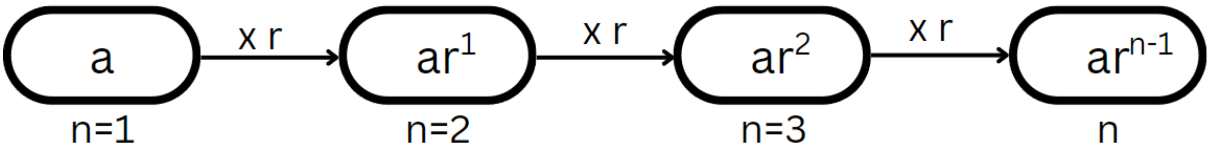
- nth term in an arithmetic progression, $\boldsymbol{t_n = a + (n-1)d}$
- Arithmetic mean in AP of three terms, a, b, c is $\boldsymbol{b = \frac{a+c}{2}}$
- Sum of n terms in AP is $\boldsymbol{S_n = \frac{n}{2}[2a + (n-1)d]}$

Geometric Progression

Geometric progression (GP) is a series of numbers where each term is obtained by multiplying or dividing a fixed quantity to its preceding term. There is a common ratio between each consecutive term.

The first term and ratio can be positive or negative, whole number, fraction, decimal values, rational or irrational.

Series	**First term (a)**	**Common ratio (r)**
2, 4, 8, 16, …	2	2
-0.5, -1.5, -4.5, …	-0.5	3
$\frac{1}{3}, \frac{5}{6}, \frac{25}{12}, \frac{125}{24}, \dots$	$\frac{1}{3}$	$\frac{5}{2}$
$\sqrt{3}, 3, 3\sqrt{3}, 9, \dots$	$\sqrt{3}$	$\sqrt{3}$



The nth term of a GP is obtained by $\boldsymbol{t_n = ar^{n-1}}$
Sum of n terms of an AP is obtained by

$$\boldsymbol{S_n = \frac{a(1-r^n)}{(1-r)}} \quad \text{if } r < 1$$

$$\boldsymbol{S_n = \frac{a(r^n-1)}{(r-1)}} \quad \text{if } r > 1$$

Geometric mean:
When three terms a, b, c are in GP, the middle term b is the geometric mean of a and c.

$$b = \sqrt{ac}$$

Example 1:
Find the 10th term in the series 3, 12, 48, … and its sum.

a = 3
r = 4

nth term $t_n = ar^{n-1}$
$t_{10} = 3 \times 4^{10-1} = 3 \times 4^9$

r > 1

Sum $S_n = \frac{a(r^n - 1)}{(r-1)}$

$$S_{10} = \frac{3(4^{10} - 1)}{4 - 1} = 4^{10} - 1$$

Example 2:

The 7th term of a series is 1/16. If ratio is ½, what is the first term, and the sum of the series from the 4th to 7th?

nth term $t_n = ar^{n-1}$

7th term $t_7 = \frac{1}{16} = a(\frac{1}{2})^{7-1} = \frac{a}{64}$

Therefore, a = 4

$S_n = \frac{a(1-r^n)}{1-r}$ as ratio is less than 1

Sum of the series from the 4th to the 7th is $S_7 - S_3$

$$S_7 - S_3 = \frac{4\left(1 - \left({}^{1}\!/_{2}\right)^7\right)}{1 - {}^{1}\!/_{2}} - \frac{4\left(1 - \left({}^{1}\!/_{2}\right)^3\right)}{1 - {}^{1}\!/_{2}} = 8\left(\frac{1}{8} - \frac{1}{128}\right) = \frac{15}{16}$$

Series is, 4, 2, 1, ½, ¼, 1/8, 1/16, 1/32, …

½+1/4+1/8+1/16 = 15/16