Gravitational Force

Gravitational force is defined as the force that pulls things down towards the earth's centre. In the universe, all objects have gravitational force and pull each other. So, just as the earth pulls us towards itself, we too pull it towards ourselves. But due to the enormous difference in masses, the effect of our pull is negligible.

Definition of Acceleration

We define acceleration as the rate of change of velocity. It means that acceleration is a change in velocity divided by time. When an object is in motion, it can change velocity due to force on it.

If the initial velocity was u, and after t seconds, it changed to velocity, v, then this means that -

$$\text{acceleration(a)} = \frac{\text{final velocity(v)} - \text{initial velocity(u)}}{\text{time(t)}}$$

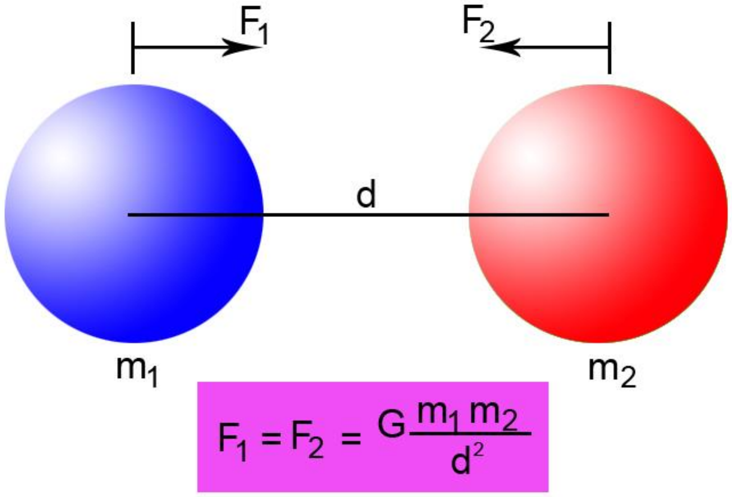
Unit of acceleration is metre per second square, $\mathrm{m/s^2}$.

What is Acceleration due to Gravity?

Acceleration due to gravity is the acceleration due to the force of gravitation of the earth. A parachutist jumping from an aeroplane falls freely for some time. He experiences an acceleration caused by the gravitational force of the earth.

The only force acting on a freely falling body is that of gravity. This force makes the things falling to increase velocity as it nears the surface. The velocity increases at a constant pace. Therefore, this acceleration is constant at all instants.

Relationship between Acceleration due to Gravity and Mass



The universal law of gravitation states that two bodies are attracted to each other by a force that is-

i. directly proportional to the product of their masses, and
ii. inversely proportional to the square of the distance d between their centres

Let us consider the mass of earth as M and an object with mass m.

$$\text{Force, F} \propto \text{M} \times \text{m}$$

$$\text{F} \propto \frac{1}{\text{d}^2}$$

Combining them,

$$\text{F} \propto \frac{\text{Mm}}{\text{d}^2}$$

$$\text{F} = \frac{\text{GMm}}{\text{d}^2}$$

G is called the universal gravitational constant and is equal to $6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$.

Force is the product of mass and acceleration.

$$\text{F} = \text{ma}$$

For gravitational force of the earth (gravity), it is $\text{F} = \text{mg}$

Therefore,

$$\text{F} = \text{mg} = \frac{\text{GMm}}{\text{d}^2}$$

$$\text{g} = \frac{\text{GM}}{\text{d}^2}$$

For an object falling towards the earth, the distance between it and the surface is negligible compared to the radius r of the earth. So, we can equate d to r.

$$g = \frac{GM}{r^2}$$

This equation shows that acceleration due to gravity, g depends on the-

i. mass of the earth, M
ii. the radius of the earth, r

The mass of the object does not affect g. All objects fall at the same acceleration simultaneously from the same height, irrespective of their weights. This was demonstrated by the Italian astronomer and scientist, Galileo Galilei from the Leaning Tower of Pisa.

If a leaf or piece of paper falls slowly, it is because of air resistance to such light objects. A parachute uses air resistance to slow down the fall. In a vacuum with no air, a feather and a block of iron fall to the ground simultaneously.

Value of Acceleration due to Gravity

Universal gravitational constant, $6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$
Mass of the earth, $M = 6 \times 10^{24}$ kg
The average radius of the earth, $r = 6.4 \times 10^{6}$ m

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 6\ x\ 10^{24}}{(6.4 \times 10^{6})^2} = 9.81\ \mathrm{m/s^2}$$

A freely falling body increases its velocity by $9.81\ \mathrm{m/s}$ every second as it nears the surface.

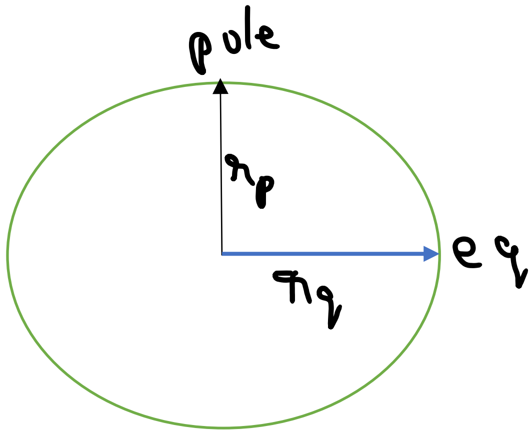
Factors affecting Acceleration due to Gravity

Three factors affect acceleration due to gravity. They are:

i. The shape of the earth.
ii. Altitude or height.
iii. Depth.

Variation in g due to the Shape of the Earth

The earth is not a perfect sphere but is bulged at the equator and slightly flattened at the poles. So, the value of g is somewhat more at the poles compared to the equator.



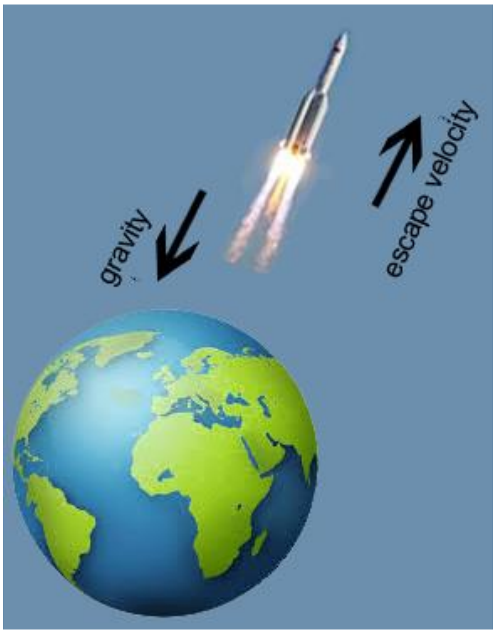
Acceleration due to gravity at the equator, $g_q = \frac{GM}{r_q^2}$

Acceleration due to gravity at the poles, $g_p = \frac{GM}{r_p^2}$

$$\frac{g_p}{g_q} = \frac{r_q^2}{r_p^2}$$

Since $r_q > r_p$, $g_p > g_q$

Escape Velocity



A ball thrown up goes up to some height and falls towards the earth. This is because of the low velocity with which it is thrown up. To move something out of the earth's gravitational force, there is a minimum velocity needed for the object to escape from its pull.

Escape velocity is defined as the minimum velocity needed by an object to overcome the earth's gravitational force. The minimum velocity with which rockets can go out into space escaping the earth's gravity is $11.205\ km/s$.

Differences between Mass and Weight

We use the terms mass and weight interchangeably in our daily lives. But in Physics, there is a difference between the two.

Mass (m)	**Weight (W)**
Mass is the amount of matter in an object.	Weight is the force by which the earth pulls an object.
It is a scalar.	It is a vector.
It remains constant throughout.	It changes according to the distance from the centre.
Mass can bever be zero.	Weight can be zero.
SI unit is kilogram (kg).	SI unit is Newton (N).

Since weight is a force, it has the same formula of force due to gravity.

$$\text{Weight}, W = mg$$

Comparison of Weight on the Earth and on the Moon

The mass of the earth is approximately 100 times that of the moon. The radius of the earth is about 4 times that of the moon.

Acceleration due to gravity for the earth:

$$g = \frac{GM}{r^2}$$

Acceleration due to gravity for the moon:

$$g_o = \frac{GM_o}{r_o^2}$$

Given,

$$M = 100M_o$$
$$r = 4r_o$$

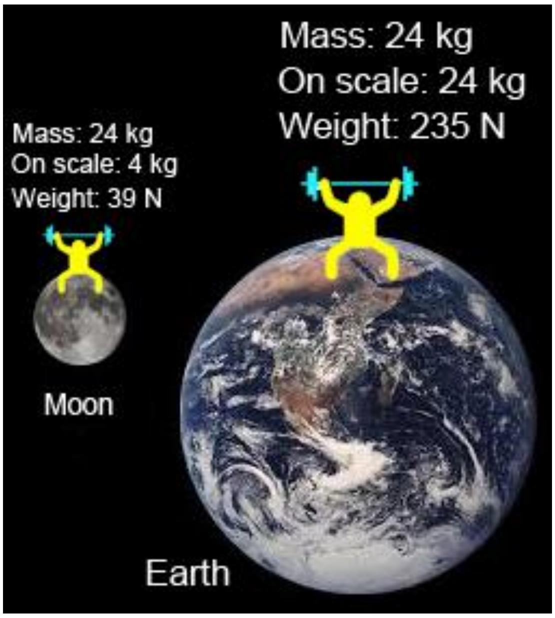
Dividing,

$$\frac{\mathrm{g}}{\mathrm{g_o}} = \frac{\mathrm{M}}{\mathrm{M_o}} \times \left(\frac{\mathrm{r_o}}{\mathrm{r}}\right)^2 = 100 \times \left(\frac{1}{4}\right)^2 = 6.25$$

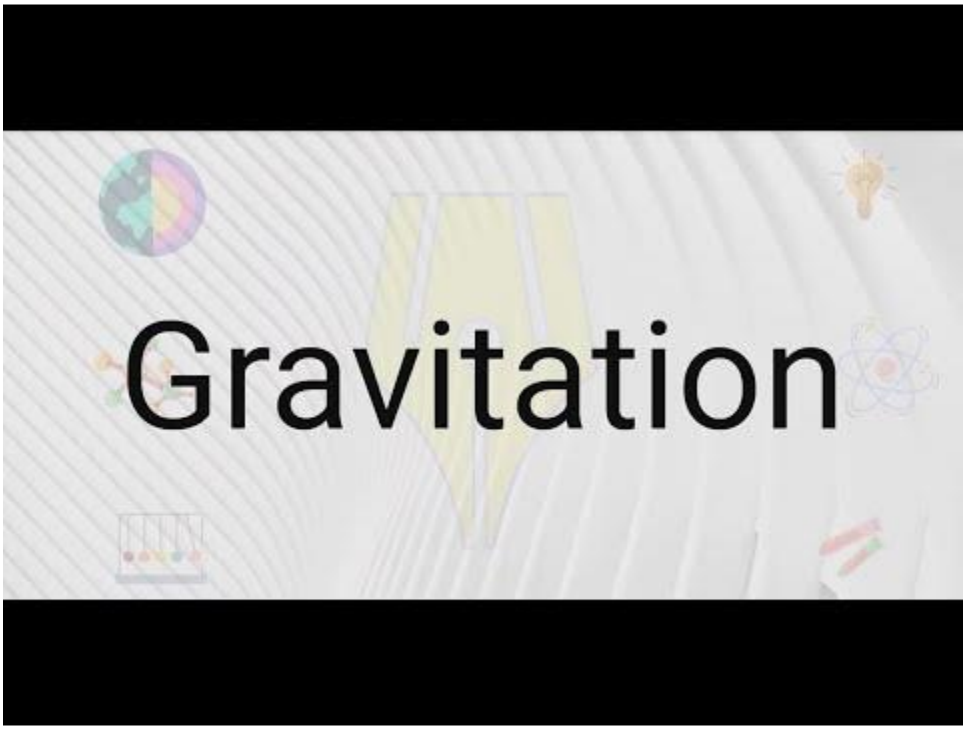
Since $weight = mg$, and m remains the same whether on earth or moon,

$$\frac{\mathrm{W}}{\mathrm{W_o}} = 6.25$$

Weight on an object is approximately 6 times that on the moon, or weight on the moon is $\frac{1}{6}th$ of that on earth.



Short video on gravitation



Summary

a. Acceleration due to gravity is constant at 9.81 m/s^2.
b. Acceleration due to gravity decreases with an increase in height above the surface.
c. Acceleration due to gravity decreases with an increase in depth below the surface.
d. The minimum velocity required for a spacecraft to move out into space is called escape velocity and is 11.2 km/s.
e. Mass is a scalar, and weight is a vector.

Sample Problems

1. The mass of the earth is 10 times that of another planet. Radius of the earth is 2 times its radius. Compare the weight of an object on the planet with that on the earth.

Solution:
Given,

$$M = 10M_p$$
$$r = 2r_p$$

Dividing,

$$\frac{\mathrm{g}}{\mathrm{g_p}} = \frac{\mathrm{M}}{\mathrm{M_p}} \times \left(\frac{\mathrm{r_p}}{\mathrm{r}}\right)^2 = 10 \times \left(\frac{1}{2}\right)^2 = 2.25$$

Since $weight = mg$, and m remains the same everywhere,

$$\frac{\mathrm{W}}{\mathrm{W_p}} = 2.25$$

Weight on the planet is approximately half of that on earth.

2. Two balls of equal mass 2 kilograms are separated by 1 metre from their centres. What is the force of attraction between them?

Solution:

$$\mathrm{F} = \frac{\mathrm{GMm}}{\mathrm{d^2}}$$

Universal gravitational constant, $G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$

$$F = \frac{6.67 \times 10^{-11} \times 2 \times 2}{1^2}$$

$$F = 2.668 \times 10^{-10}\ \text{Newtons}$$

Frequently Asked Questions (FAQs)

1. Do the earth and moon attract each other?

Ans: Yes, any two objects in space attract each other. Due to the larger mass, the earth's force of attraction is more than that of the moon.

2. Where is the acceleration due to gravity more, at the poles or at the equator?

Ans: Acceleration due to gravity is inversely proportional to the square of the distance between the centre and the surface. Poles are closer to the centre than the equator. So, acceleration due to gravity is more at the poles.

3. Is the gravitational force of attraction of the earth applicable for infinite distance?

Ans: No. Acceleration due to gravity decreases with an increase in height. At a certain height, this is negligibly small and almost zero.

4. Why do we say that the unit of weight is Newton?

Ans: Weight is the force due to the mass acting downwards. So, its unit is Newton.

5. Why cannot aeroplanes be used to go into outer space?

Ans: A minimum velocity is required to escape from earth's gravitational force, called escape velocity. Aeroplanes cannot travel at this velocity.