Principle Of Conservation Of Momentum – Derivation, Examples

Conservation of Momentum is one of the basic concepts of physics. The term conservation gives us an idea of an unchanging quantity. What is momentum, and how is it conserved? We shall see this now.

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The Principle Of Conservation Of Momentum

The principle of conservation of momentum states that *the total momentum in a closed system is constant unless an external force is applied*.

What Is Momentum?

We define the momentum (p) of an object as the product of its mass (m) and its velocity (v).

$$p = mv$$

The SI unit for momentum is $\text{kg} - \text{m/s}$. (kgms^{-1})

Momentum is a vector, meaning that it has a magnitude (numerical value) and a direction due to velocity, a vector. Momentum and its conservation are related to Newton's second and third laws of motion.

Newton's Second Law of Motion

According to this law, acceleration (a) of an object is,

i. directly proportional to the force (F) acting on it, and
ii. inversely proportional to the mass (m) of the object

$$a = \frac{F}{m}$$

Therefore, force is the product of mass and acceleration.
Acceleration is the ratio of change in velocity to time.

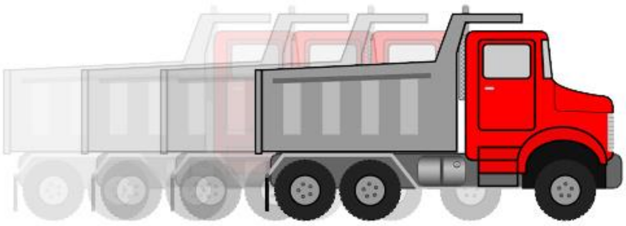
$$F = ma = m\frac{v}{t}$$

So, we get,

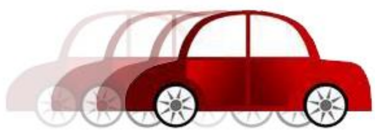
$$mv = Ft$$

Change in momentum is the product of force and the time interval. Therefore, we can also write momentum as Ns unit.

Using this relation, we can write Newton's second law in different ways, as:
i. rate of change of momentum is equal to the force acting on the object
ii. the momentum of some object is the time required for a constant force to bring it to rest



More mass, more time to stop



Less mass, less time to stop

Newton's second law deals with the definition of momentum. The third law deals with the conservation of momentum.

Newton's Third Law of Motion
It states that for every action, there is an equal and opposite action.

The law of conservation of momentum is derived from Newton's third law. An object X applies force on object Y. Therefore; object Y applies an equal amount of force on X in the opposite direction.

Assuming there is no external force, a change in momentum in one body is balanced with an equal and opposite change in momentum in the other body. **This means that the total momentum in the system does not change.** This is the principle of conservation of momentum.

In a system with many objects, the total momentum is the sum of individual momenta. Since momentum is a vector, the momentum of different objects in different directions can cancel each other, giving the overall sum as zero.

Derivation Of The Law Of Conservation Of Momentum

Let there be two objects A and B that collide with each other.

For object A, initial and final velocities are $u_A$and v_A, mass is m_A
For object B, initial and final velocities are $u_B$and v_B, mass is m_B

From Newton's second law, force is given by,

$$F = ma = m\frac{(v-u)}{t}$$

Force exerted by A on B is,

$$F_A = \frac{m_A(v_A - u_A)}{t_A}$$

Force exerted by B on A is,

$$F_B = \frac{m_B(v_B - u_B)}{t_B}$$

where,
t_A and t_b are the time taken by the objects to stop after colliding.
However, both these times are equal. So, $t_A = t_B = t$

From Newton's third law, these two forces are equal and opposite.

$$F_A = -F_B$$

$$\frac{m_A(v_A - u_A)}{t} = -\frac{m_B(v_B - u_B)}{t}$$

$$m_A(v_A - u_A) = -m_B(v_B - u_B)$$

$$m_A v_A - m_A u_A = -m_B v_B + m_B u_B$$

Rearranging the terms,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

The above equation shows that the **sum of momentum before the collision is equal to the sum of momentum after the collision**. This means that overall momentum in the system is the same, or momentum is conserved.

In a closed system with multiple objects or particles, this can be generalized as,

$$m_1u_1 + m_2u_2 + m_3u_3 + \cdots + m_nu_n = m_1v_1 + m_2v_2 + m_3v_3 + \cdots + m_nv_n$$

or

$$p_{before} = p_{after}$$

Types Of Collisions

Collision happens when two or more moving objects make physical contact at some velocity. There are three types of collisions:

Perfectly elastic collisions:
Gas molecules experience perfectly elastic collisions. Here, the velocity before and after collision does not change.

Perfectly inelastic collisions:
They happen when the moving object or objects stick together after hitting each other. Velocity after the collision is zero. A ball of dough getting stuck to a wall after being thrown at it can be an example of this.

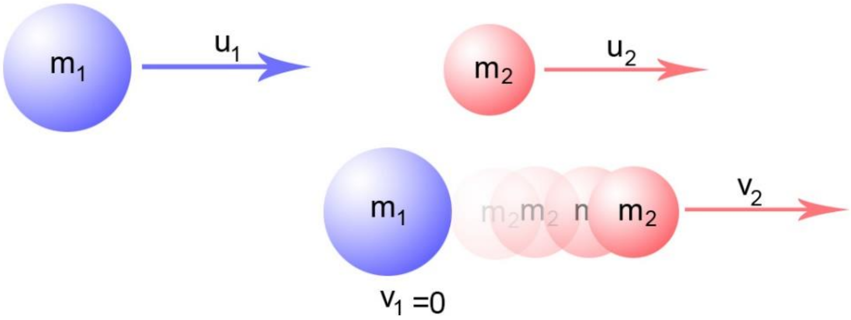
Inelastic collisions:
It is a case between elastic and inelastic collisions. The bodies on colliding move in a different direction but with a lesser velocity.

Momentum For Some Types Of Collisions

Let us see a few common ways when two objects make physical contact with each other.

Let initial velocities of both objects be u_1 and u_2 before the collision.
Let final velocities be v_1 and v_2 after the collision.
Let the masses of both objects be m_1 and m_2.

1. Two billiards balls moving in the same direction collide.



Ball 1 and 2 are moving in the same direction. After hitting ball 2, ball 1 stops.

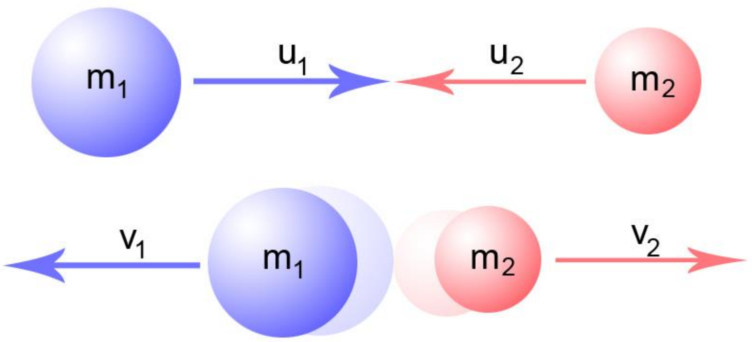
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Since $v_1 = 0$,

$$m_1u_1 + m_2u_2 = m_2v_2$$

The entire momentum is transferred to the second ball.

2. The two balls collide head-on.



Ball 1 and 2 are moving in the opposite direction. Once they hit each other, the balls move in the direction opposite to their original direction.

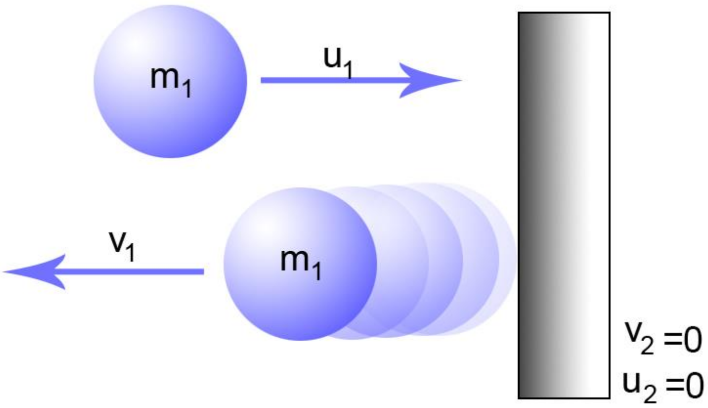
Considering right as positive and left as negative,

$$m_1u_1 - m_2u_2 = -m_1v_1 + m_2v_2$$

$$m_1u_1 + m_1v_1 = m_2u_2 + m_2v_2$$

The sum of the momentum of ball 1 before and after the collision is equal to the sum of the momentum of ball 2 before and after the collision.

3. The billiards ball hits the side of the table perpendicularly.



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

The sides of the table are not moving, so both u_2 and v_2 are zero. Since the ball changes direction opposite to its original, v_1 is negative.

$$m_1u_1 + m_2u_2 = -m_1v_1 + m_2v_2$$

$$m_1u_1 = -m_1v_1$$

Therefore,

$$u_1 = -v_1$$

This is an elastic collision as the ball returns back with the same velocity as the original.
Change in momentum is $p_{final} - p_{initial}$
Therefore,

$$change\ in\ momentum = m_1v_1 - m_1u_1$$

Since,

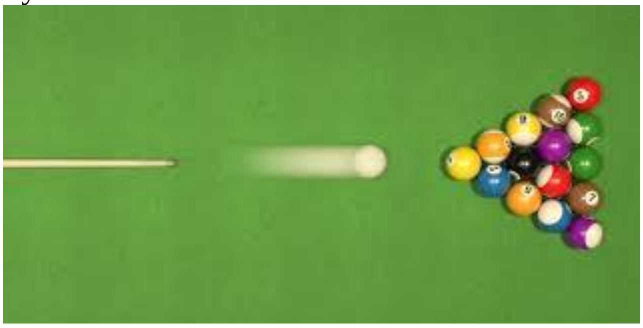
$$v_1 = -u_1$$

$$change\ in\ momentum = -m_1u_1 - m_1u_1 = -2m_1u_1$$

Momentum has changed to twice the original momentum in the opposite direction of movement. If momentum is conserved, how did it double? It means that the same momentum is seen in the table side also, but being unmoving, it is not noticed.

Examples Of Conservation Of Momentum

a. A ball hits other balls and scatters them in all directions. However, the momentum lost by the hitting ball is gained by the other balls causing them to move. Thus, the total momentum in this system remains the same.



b. A gun recoils when a bullet is fired. Initially, there is no movement, so the sum of momentum due to bullet and gun is zero. From conservation law, the final sum of momentum must be zero.

$$m_bu_b + m_gu_g = m_bv_b + m_gv_g$$

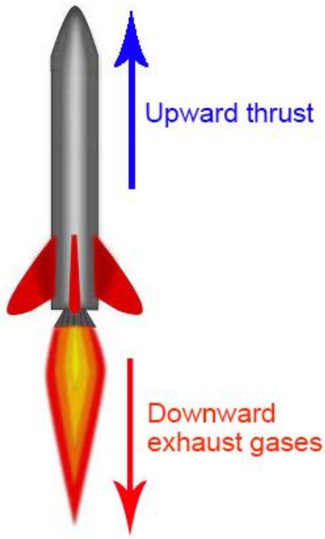
Initially, both gun and bullet are at rest, so $u_b = 0$ and $u_g = 0$

$$0 = m_bv_b + m_gv_g$$

Therefore, the velocity of gun recoil is,

$$v_g = -\frac{m_b}{m_g}v_b$$

The mass of the bullet is negligible compared to the mass of the gun. Therefore, recoil velocity is minimal compared to the velocity of the bullet. The negative sign shows that the gun recoils in the opposite direction to the movement of the bullet.



Conservation Of Momentum – Sample Problem

1. A 10 gm bullet is fired from a gun with a velocity of 600 m/s. The recoil velocity of the gun is 1 m/s. What is the mass of the gun?

Solution:
Given,
The velocity of the bullet, $v_b = 600 \text{ m/s}$
Recoil velocity of the gun, $v_g = 1 \text{ m/s}$
Mass of the bullet, $m_b = 10 \text{ gm} = 0.01 \text{ kg}$

The velocity of gun recoil is,

$$v_g = -\frac{m_b}{m_g} v_b$$

$$1 = -\frac{0.01}{m_g} \times 600$$

Mass cannot be negative. Therefore, we remove the negative sign from the equation.
Solving, the mass of the gun is,

$$m_g = 6 \text{ kg}$$

2. A bat of 300 gm hits a ball of 75 gm fallen on the ground at a velocity of 60 m/s. The ball moves at the same velocity. What is the velocity of the bat after hitting the ball?

Solution:
Given,
Mass of the bat, $m_1 = 300 \text{ g} = 0.3 \text{ kg}$
Mass of the ball, $m_2 = 75 \text{ gm} = 0.075 \text{ kg}$
Initial velocity of the bat, $u_1 = 60 \text{ m/s}$
Initial velocity of the ball, $u_2 = 0 \text{ m/s}$
Final velocity of the ball, $v_2 = 60 \text{ m/s}$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0.3 \times 60 + 0 = 0.3 \times v_1 + 0.075 \times 60$$

Solving for v_1,

The velocity of the bat after hitting the ball is $45\ \mathrm{m/s}$.

FAQs On Conservation Of Momentum

1. What kind of collision is the collision between molecules of air?

Ans: It is a perfectly elastic collision when air molecules collide.

2. What causes rockets to move up when gases come out of its engine?

Ans: The downward rush of gases at a very high velocity has a high momentum. This creates an equal momentum in the upward direction pushing the rocket up.

3. Why is momentum a vector quantity?

Ans: Momentum is proportional to velocity, which is a vector.

4. Why does it take some time for a speeding vehicle to come to rest when brakes are applied suddenly?

Ans: The momentum of an object is the time required for a constant force to bring it to rest. The higher the speed, the higher is its momentum. More time is needed for it to stop completely.

5. Why is it easy to stop a car suddenly compared to a truck with both running at the same velocity?

Ans: Momentum is proportional to the mass of the moving object. A car is lighter compared to a truck, so it has lesser momentum. Time and the force needed to stop it is less.